Abstract

Welcome to a journey through math and physics. In this book a number of formulas will be discussed and explained. A formula represents much more than a couple of math symbols, it represents concepts, ideas, and intuition.

In my humble opinion formulas and equations are the gateway to deeper understanding. They can be powerful, magical and baffling. I hope you are willing to take this journey with me.

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Elaborate online material for all the formulas can be found here: `http://www.bozon.org/`

$$E = hf$$

Energy of a photon

For the start of this story we need to go quite a long way back. In 1803 Young performed an experiment in which he shone light (of one single color) on a screen containing two small slits. The results were surprising. Instead of two bright spots he observed several spots.

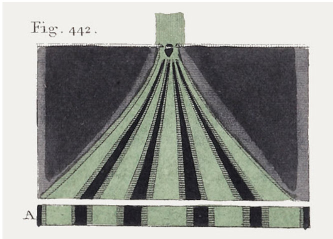

Figure 1: Double slit experiment by Thomas Young [1].

This (and other supporting experiments) settled a debate between Isaac Newton and Christiaan Huygens. Both had conflicting answers for the question: what does light consist of? Newton assumed that light was made up of particles. Huygens was convinced it had a wavelike character. The wave like character could explain the results of Young's experiment. The two slits would both generate waves. The crest of one wave would combine with the crest of the other wave to make a bright spot. If however the trough of one wave and the crest of the other wave would combine the waves would cancel each other out. This would give a dark spot. The pattern thus generated is called a interference pattern:

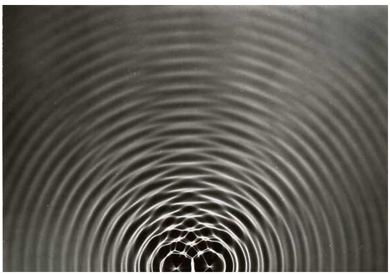

Figure 2: Interference pattern by Bernice Abbott [2].

The interference pattern was clearly visible in Young's experiment and thus proved the Dutchman Huygens right. This also led to the discovery that light has a wavelength (the wavelength is equal to the distance between two crests of the wave). Blue light has a short wavelength, red light a long wavelength. There was also an indication that the waves with shorter wavelengths had more energy (sunburn is mostly caused by UV light, which has a short wavelength).

Fast forward to approximately 100 years later, the photoelectric effect experiment. In this experiment, a beam of light (of different colors, and thus different wavelengths) was aimed at a metal plate. The idea was that the energy of the light could knock electrons out of their orbit. However, the measurements showed that only for certain light colors electrons were released from the metal. The prevailing thought at that time was that the

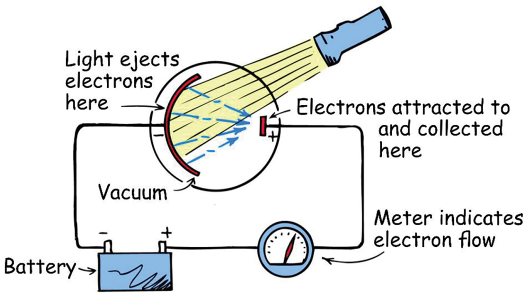

Figure 3: The photoelectric effect [3].

energy of light was also proportional to the height of the waves of the light. So the higher the wave the more energy. So even a wave with a long wavelength should be able to 'free' the electrons with its energy if just more light was shone on it. Apparently this was not the case!

The solution was assuming light (of one color) is not one big wave, but several small waves (all the same height) with the same wavelength. Apparently only one of those waves could at the same time interact with the electron. This was the birth of the photon idea (a 'particle' of light). It seemed that Newton was also slightly right, the light waves come in multiple small parts which we now call photons.

Figure 4: Left the classical view, right the idea of photons.

The wavelength, λ, of a photon can be transformed to the frequency of the photon:

$$f = \frac{\lambda}{c}$$

with c being the speed of light. Back to our equation:

$$E = hf$$

h, the Planck constant, will play a big role in the next chapters. For now, just remember it is a constant (6.626×10^{-34} J.Hz^{-1}). So the energy of a photon is equal to it's frequency (how often it "vibrates" per second) times a very small constant. Why is this formula so interesting? It turned out that this was the first small step towards the idea that all things (particles) are actually vibrating fields, which is the basic idea of quantum mechanics.

$$e^{i\pi} = -1$$

Euler's identity

This equation starts with a strange idea, what if the root of a negative number existed? Its a little bit difficult to imagine, but lets go along with the idea. For this we first introduce a new number, i, with $i^2 = -1$. We call this an imaginary number (very appropriate name). Combining imaginary numbers with normal ('real') numbers we can create complex numbers:

$$a + bi$$

We can plot this number on a 2-dimensional plane:

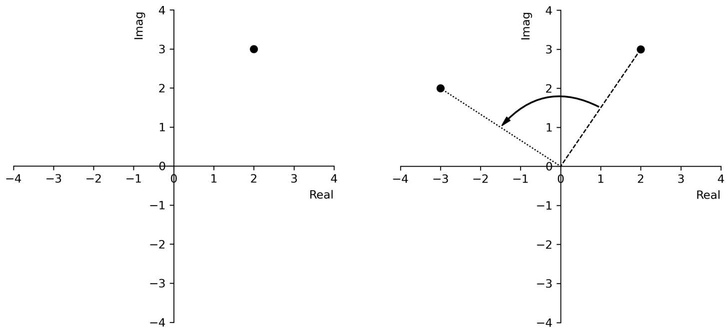

Figure 5: Left, the number $2 + 3i$, right the same number multiplied by i, $i(2 + 3i) = -3 + 2i$.

In figure 5 we see that multiplying by i equals rotating counterclockwise 90° around the origin. Apparently there is a connection between circular motion and complex numbers.

We will now take a little side trip to differential equations and exponential functions. The solution for the following differential equation:

$$\frac{df(t)}{dt} = f(t)$$

is given by:

$$f(t) = c.e^t$$