THEORETICAL PHYSICS

Unthinkable Works!

by

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Cover: Simple pendulum engraved on an illustration of an atomic orbital. Upright figures signify "evolution" for representing birth-and-death.

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Preface

I began writing this monograph under the title "Theoretical Physics: Unthinkable Works!", only to find that the monograph had fallen short of making any kind of serious impact, despite my earlier attempts with various independent publishers. This was due to the quality of earlier editions lacking "motivation" or proper "explanations" in many parts of each chapter. Understandably, I was rushed into writing the first monograph during a period when I didn't have an academic appointment and, in effect, was without any possibility of receiving funding for independent research.

The chapters in this edition of the monograph are essentially the same as the chapters presented in the earlier editions, since they grew out of the research notes that I had originally "sketched" out – around Christmas of the year 2007 – in my own dwelling in Australia. These notes were, then, further developed into a series of preprints in the year 2008, while I was affiliated with the Institute of Mathematical Sciences at the University of Malaya. The works of this monograph have once again been embodied as a "three-of-a-kind" collection of essays, each of which overcame insurmountable obstacles to offer precise and analytical solutions to theoretical physics problems that looked unsolvable.

Chapter 1 begins with a revisit of the simple harmonic oscillator as first solved by Erwin Schrödinger. Here the reader learns of a serious *limit* that is disobeyed by the "exact" wave function, which is perceived as a severe flaw of the oscillator's wave function in wave mechanics. Consequently, **Chapter 1** of this monograph is whole-heartedly engrossed in a "quest" to rectify this observable flaw as well as to formulate a complete wave function that reveals the "true" wave function of the harmonic oscillator.

In **Chapter 2**, I have decided to take on the Markovian birth-and-death differential-difference equations. This is a special kind of stochastic equation that not only brings a special charm to merging physics with the natural laws of ecology, but has a founding significance in the *theory of evolution*. If this is true, then surely the exact solutions should possess an evolutionary life-cycle that is periodic in nature? To meet this expectation, of course, requires getting through an impregnably difficult and *infinite sequence* of difference equations before establishing uniqueness with any exact solution of the birth-death equations. This has placed a further demand in **Chapter 2** to expose a somewhat unexpected symmetry principle, known as *anti-symmetrization*, which emerges in the chapter as a "hidden-symmetry" in the birth-death equations.

Notorious from the legendary observations of Galileo Galilei around 1590, the *simple pendulum* has preoccupied the best scientific and mathematical minds to such a high degree that this became a period when the *mathematical theory of elliptic functions* was borne. My own recollections of learning about the theory of elliptic functions is one of dissatisfaction, since I felt the theory to be too artificial an "invention" for solving the equations of motion of the simple pendulum. Devoted to solving the simple pendulum problem, which is over four-hundred years old, is, therefore, the penultimate challenge of **Chapter 3**.

I am, of course, indebted to the University of Malaya for their kindness and hospitality, although it should be mentioned that no part of the research in this monograph was conducted with the support of any type of grant or financial aid.

I am also admitting here a very disheartening moment: the recent passing of my pet Chihuahua, "Zinga". He was often regarded, at times, as practically a "fellow colleague" of such fortitude and sort, and became my most inspirational companion throughout the ensuing years of this research. Last to be mentioned is the imprint on the inside of the title page, which has been designed as a symbol of meeting life's toughest challenges head-on, represented by the "rhinoceros and its horns". This symbol is also a reminder that the rhinoceros, amongst many of the other wildlife species inhabiting this planet, is truly an endangered species that needs to be protected, so that *proceeds* from this monograph are intended as a means of raising funds for such a cause.

Derek J. Daniel
December, 2022

Contents

Chapter 1

The Quest of Finding the Missing Solutions for the Quantum Mechanical Oscillator

The issue of the angular frequency ω in Erwin Schrödinger's exact wave function for the simple harmonic oscillator approaching a zero limit, i.e., $\omega \to 0$, is a matter of serious concern in Quantum Mechanics. This is because in the limit $\omega \to 0$, the "true" solution for the simple harmonic oscillator should reduce to that of a *plane wave*. The main objective of this chapter, therefore, is to incorporate the correct limiting behavior in the wave function of the simple harmonic oscillator, inferring that previous treatment of the oscillator in Quantum Mechanics cannot possibly be correct!

1.1 Introduction

Schrödinger's time-independent equation for the quantum mechanical oscillator, also called the *linear* or *simple harmonic oscillator*, is defined by

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+\tfrac{1}{2}m\omega^2x^2\psi=E\psi, \tag{1}$$

which, with a slight change in notation, coincides with the definition used in **Chapter 4** by Schiff (1965) [1], where ψ is the wave function, ω is the angular frequency of the oscillator, E is its energy, and h is Planck's constant which is absorbed into $\hbar = h/2\pi$.

The wave function solution ψ to Eq. (1) is celebrated in pp. 62 of Schiff (1965) [1] by an orthogonal class of polynomials,

$$\psi = e^{-\frac{m\omega}{2\hbar}x^2} H_n\left(\sqrt{m\omega/\hbar}\,x\right), \tag{2}$$

called the *Hermite polynomials* of degree n, as denoted by $H_n\left(\sqrt{m\omega/\hbar}\,x\right)$ in Eq. (2) (without normalization). However, the surprising instance with this set of polynomials is when the angular frequency ω of the linear harmonic oscillator approaches zero, in which case, Eq. (2) becomes

$$\lim_{\omega\to 0}\psi = \text{constant},$$

If the same limit, $\omega \to 0$, is applied to the Schrödinger wave equation in Eq. (1), the resulting wave function ψ is easily verified to be a plane wave

$$\psi \sim e^{ikx},$$

whose wave-number k is determined by $k = \sqrt{2mE/\hbar^2}$. This result is slightly more alarming than the previous one, since it implies the true solution should unquestionably be that of a plane wave.

Consequently, the original derivation presented in **Chapter 4** of Schiff (1965) [1] will be cast aside, for now, and a total reformulation of the wave function ψ, governed by the Schrödinger's wave equation in Eq. (1), will be performed in the next section. In doing so, it is important to keep in mind of the fact that ψ must reduce to its natural plane wave state, $\psi \sim e^{ikx}$, whenever the limit $\omega \to 0$ is invoked, so that this limit behaviour will necessarily need to be incorporated into ψ when seeking a *bona fide* wave function.

1.2 Reformulation of the Wave Function $\psi(z)$

§. ***The Wave function Ansatz.*** The Schrödinger wave equation defined by Eq. (1) can be rewritten, in dimensionless form,

$$\frac{d^2\psi(z)}{dz^2} + \left(\varepsilon - \tfrac{1}{4}z^2\right)\psi(z) = 0, \tag{3}$$

where z is made explicit here in the argument of the wave function $\psi(z)$ through the dimensionless quantities,

$$x = \tfrac{1}{\sqrt{2}}az, \quad a = \sqrt{\frac{\hbar}{m\omega}}, \quad \text{and} \quad \varepsilon = \frac{mEa^2}{\hbar^2} = \frac{mE}{\hbar^2}\frac{\hbar}{m\omega} = \frac{E}{\hbar\omega}.$$